Reply to the comment on "On the problem of initial conditions in cosmological N-body simulations"

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In [1] the authors criticize our measurements [2] of the reduced two-point correlation function $\xi(r)$ and of the one-point number variance $\sigma^2(r)$. On one hand, they confirm our conclusion that the estimator of $\xi(r)$ for the initial conditions (IC) is in *disagreement* with the theoretical prediction at any scale in the simulation. However, the authors ascribe such a disagreement to the estimator's noise which they claim to be larger than the expected signal at any scale in the simulation. On the other hand, they find that the behavior of the estimator of $\sigma^2(r)$ is in good agreement with the theoretical expected behavior. Note that they do not provide any argument to explain the mismatch between the behaviours of $\xi(r)$ and $\sigma^2(r)$. In order to clarify the matter we have firstly recomputed both estimators and we have increased the statistical precision. Our result is that the new estimation of $\xi(r)$ is in agreement with our previous one and with the one by [1], while the new $\sigma^2(r)$ is in a better agreement with the theoretical behavior (see Fig.1). Hence, at least from the point of view of estimations, our conclusions are in good agreement with those of [1]. Our previous determination of $\sigma^2(r)$ was affected by an imprecision in the normalization of the box which affected our estimation at scales comparable to the box size. The main point of disagreement with [1] concerns the interpretation of the measured statistical estimators, and their relation with the expected theoretical behaviors. The following discussion will allow us to clarify also the mismatch in the determinations of $\sigma^2(r)$.

The estimators of $\sigma^2(r)$ and $\xi(r)$ are directly related (fluctuations in number of points in balls or shells centered on a random or distribution point normalized to the mean density [3]) and there is a precise theoretical relation with links these two statistics for a distribution with average density n_0:

$$\sigma^2(r) = \frac{1}{V(r)^2}\int_V d^3x \int_V d^3y \,\tilde{\xi}(|\vec{x}-\vec{y}|) = \frac{1}{n_0 V(r)} + \frac{1}{V(r)^2}\int_V d^3x \int_V d^3y \,\xi(|\vec{x}-\vec{y}|)\,, \quad (1)$$

where the second equality is explicitly satisfied by *any* discrete distribution; we have defined $\xi(r)$ to be the non diagonal part of the complete correlation function $\tilde{\xi}(r) = \delta(r)/n_0 + \xi(r)$ and hereafter we refer to the first term on the right hand side of Eq.1 as the "discrete term" while the second part is denominated "correlated term". Note that in the simulation we estimate the non diagonal part of the correlation function $\xi_S(r)$ (while the theoretical model

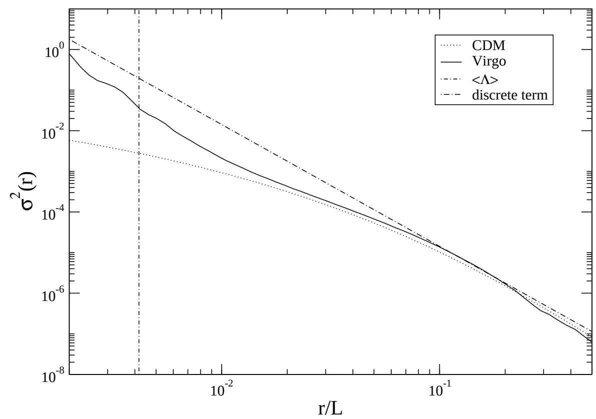

Fig. 1 – Normalized mass variance for the IC (Virgo), theoretical behavior (CDM) and the contribution of the "discrete term". The average distance between nearest neighbors $\langle\Lambda\rangle$ is also reported.

gives the prediction for the complete $\tilde{\xi}_T(r)$ of a continuous field), and the estimator of $\sigma^2(r)$ contains both the discrete and correlated term (see [3] for more details). Now it is clear that, if the variance in the simulation $\sigma_S^2(r)$ agrees with the theoretical one $\sigma_T^2(r)$ we expect that $\tilde{\xi}_T(r) \simeq \xi_S(r)$ *if* the discrete term in Eq.1. is negligible with respect to the correlated term. However this is not the case: the contribution of $1/n_0 V$ is of order $\sigma_T^2(r)$ at scales a few times larger than the mean interparticle distance $\langle\Lambda\rangle$ (see Fig.1). Hence our conclusion is that the agreement of the estimator of $\sigma^2(r)$ with $\sigma_T^2(r)$ is due to the fact that the particle distribution has still an average density n_0 too small to have a mass variance dominated by the "correlated term" and discreteness effects are dominant at scale larger than 1/10th of the box size.

This is in agreement with the result that the behavior of the estimated $\xi_S(r)$ is different at all scales from the expected one of $\tilde{\xi}_T(r)$: Its amplitude is very small on large scales and it becomes negligible with respect to discrete noise. A simple estimation of such a noise, by taking the *lower limit* of the Poisson case (i.e. the discrete term only), gives that $\delta\xi \approx 1/\sqrt{n_0} \approx 1/\sqrt{256^3} \approx 2 \times 10^{-4}$ (as $V = 1$ in our units). Using $\tilde{\xi}_T(r)$ one can see that $|\tilde{\xi}_T| < \delta\xi$ for $r \gtrsim 0.05 \approx 10\langle\Lambda\rangle$. Now, given the fact that at scales smaller than $3\langle\Lambda\rangle \approx 0.015$ ξ_S is still oscillating between positive and negative values, due to the in-print of the pre-initial distribution, at best one can recover $\tilde{\xi}_T$ in the range $\approx [3, 10] \times \langle\Lambda\rangle$, while on larger scales the "discrete" noise predominates. Clearly such an agreement at intermediate scales is verified if there is not, in addition to "discrete" noise, a *systematic difference* between $\tilde{\xi}_T$ and ξ_S: instead this seems to be the case as $\xi_S \neq \tilde{\xi}_T$ at all scales. For the power spectrum (which we have not discussed in our paper) the situation is rather similar to the case of the mass variance as $P(k)k^3 \approx \sigma^2(r = 1/k)$ (but see [3]). We believe that a more complete theoretical understanding of the important problem of generating particle distribution with given correlations must be addressed by computing the effect of the application of a correlated displacement field on a generic particle distribution: We will discuss such a point in a forthcoming paper.

REFERENCES

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